v.imp

Beats and beat frequency :→

when two sound waves of same amplitude but slightly different frequency move in the same direction then the waves superimpose to each other giving rise to alternating hearing of the sound. This phenomenon is called beats.

The number of beats formed per second is called beat frequency.

Expression of beat frequency :→

Suppose, two waves of frequencies f_1 and f_2 travelling in the same direction superimpose on each other. The equations of two waves are given by,

$$y_1 = a \sin w_1 t \quad \text{——} (1)$$

$$y_2 = a \sin w_2 t \quad \text{——} (2)$$

where, a is amplitude.

since, $w = 2\pi f$;

Then, $y_1 = a \sin 2\pi f_1 t$ ——(3)

& $y_2 = a \sin 2\pi f_2 t$ ——(4)

So, the resultant displacement is,

$$y = y_1 + y_2$$

$$\therefore y = a \sin 2\pi f_1 t + a \sin 2\pi f_2 t$$

$$= a \left[\sin 2\pi f_1 t + \sin 2\pi f_2 t \right]$$

$$\therefore y = a \cdot 2\sin\left(\frac{2\pi f_1 t + 2\pi f_2 t}{2}\right) \cdot \cos\left(\frac{2\pi f_1 t - 2\pi f_2 t}{2}\right) \quad \text{②}$$

$$\left[\because \sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right)\right]$$

$$\therefore y = 2a\cos\left[\frac{2\pi \cdot (f_1 - f_2) \cdot t}{2}\right] \cdot \sin\left[\frac{2\pi \cdot (f_1 + f_2) t}{2}\right]$$

$$\therefore y = A \sin 2\pi \left(\frac{f_1 + f_2}{2}\right) \cdot t \quad \text{——— ⑤}$$

where, $A = 2a\cos\left[\frac{2\pi (f_1 - f_2) t}{2}\right]$ is the resultant amplitude

Now, the amplitude (A) will be maximum,

if $\cos\left[\frac{2\pi (f_1 - f_2) t}{2}\right] = \pm 1$

or, $\cos\left[\frac{2\pi (f_1 - f_2) t}{2}\right] = \cos n\pi$; where, $n = 0, 1, 2, \cdots$

$$\therefore \frac{2\pi (f_1 - f_2) t}{2} = n\pi$$

$$\therefore (f_1 - f_2) t = n$$

$$\therefore t = \frac{n}{f_1 - f_2} \quad \text{——— ⑥}$$

putting values of n then from eqn ⑥, we get,

$$\therefore t = 0, \frac{1}{f_1 - f_2}, \frac{2}{f_1 - f_2}, \cdots$$

i.e. Time period $(T) = \frac{1}{f_1 - f_2}$

$$\therefore \boxed{\text{beat frequency}(f) = \frac{1}{T} = f_1 - f_2} \quad //$$